Skm's J. M. Patel College of Commerce

Programme: B. Sc. I. T.

Course: Applied Mathematics

Topic: Matrices

Matrices:

An arrangements of an element in particular rows and columns is known as matrix

Where elements of matrix are enclosed in parenthesis ( ) or [ ].

The matrix is denoted by upper case letters.

OR :

An array of dimension m × n is known as Matrix
where m is number of rows (horizontally lines) and n is number of column (vertical lines)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \\ -5 & 0 \end{pmatrix} \qquad X = \begin{bmatrix} -2 & 3 & 5 \\ 8 & 0 & 4 \\ 12 & 52 & 7 \end{bmatrix}$$

Let A= [a_{ij}]$_{m\times n}$ where i = 1 , 2, 3, ... m and j= 1, 2, 3, ...n

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2(n-1)} & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3(n-1)} & a_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{(m-1)1} & a_{(m-1)2} & \cdots & a_{(m-1)(n-1)} & a_{(m-1)n} \\ a_{m1} & a_{m2} & \cdots & a_{m(n-1)} & a_{mn} \end{bmatrix}$$

Note:

If m $\neq$ n then matrix A is Rectangular matrix.

If m = n then matrix A is Square matrix.

$$A=\begin{array}{cl}\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2(n-1)} & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3(n-1)} & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{(m-1)1} & a_{(m-1)2} & \cdots & a_{(m-1)(n-1)} & a_{(m-1)n} \\ a_{m1} & a_{m2} & \cdots & a_{m(n-1)} & a_{mn} \end{bmatrix} & \begin{array}{l} \rightarrow R_1 : \text{first row} \\ \rightarrow R_2 : \text{second row} \\ \rightarrow R_3 : \text{third row} \\ \\ \rightarrow R_{m-1} : (m-1)^{th} \text{ row} \\ \rightarrow R_m : m^{th} \text{ row} \end{array}\end{array}$$

- A(i, j): a_{ij} : elemment present at ith row and jth column

- Order of matrix is $m \times n$

1 Rectangular Matrix

A matrix of order m × n is known as Rectangular matrix.

(Number of rows and column are distinct

i.e. $m \neq n$.)

$$A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \\ -5 & 0 \end{pmatrix}$$

Number of rows= 3
Number of column = 2
Order of matrix A is 3×2

$$X = \begin{pmatrix} p & q & r \\ s & t & u \end{pmatrix}$$

Number of rows= 2
Number of column = 3
Order of matrix X is 2×3

2 Square Matrix

A matrix of order m × n where **m = n** is known as Square matrix.

(Number of rows and number of columns are same)

$$X = \begin{bmatrix} -2 & 3 & 5 \\ 8 & 0 & 4 \\ 12 & 52 & 7 \end{bmatrix}$$

Number of rows= 3
Number of column = 3
Order of matrix X is 3×3

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Number of rows= 2
Number of column = 2
Order of matrix A is 2×2

3 Row Matrix

A matrix having only one row irrespective of number of column is known as Row Matrix.

$P = \begin{pmatrix} 5 & 7 \end{pmatrix}$

Number of rows= 1
Number of column = 2
Order of matrix P is 1×2

$Q = \begin{bmatrix} 10 & 25 & -15 & 87 \end{bmatrix}$

Number of rows= 1
Number of column = 4
Order of matrix Q is 1×4

4 Column Matrix

A matrix having only one column irrespective of number of rows is known as Column Matrix.

$S = \begin{pmatrix} 4 \\ 5 \\ 4 \end{pmatrix}$

Number of rows= 3
Number of column = 1
Order of matrix S is 3×1

$T = \begin{pmatrix} 3 \\ 5 \\ 8 \\ 0 \\ 20 \end{pmatrix}$

Number of rows= 5
Number of column = 1
Order of matrix T is 5×1

5 Null matrix / Zero Matrix

A matrix whose all elements are zero ' 0 ' is known as Null Matrix or Zero Matrix.

$D = \begin{pmatrix} 0 & 0 \end{pmatrix}$

Order(D)= 1×2

$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Order(A)= 3×3

6 Diagonal Matrix

A square matrix in which principal diagonal elements are non zero and rest of the elements are zero is known as Diagonal matrix.

OR

Let A= $[\, a_{ij} \,]_{m\times m}$ where i , j = 1 , 2, 3, ... m.

A is said to be diagonal matrix

If $a_{ij} = 0 \ \forall\, i \neq j$

$a_{ii} \neq 0$ at least for some i

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

$$D=\begin{pmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$D=\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$D=\begin{pmatrix} 1 & 0 \\ 0 & 20 \end{pmatrix}$$

NOTE:

i. Matrix is square

ii. All entries above the diagonal and below the diagonal must be zero ‘ 0 ’.

iii. Principal diagonal entries are non zero.

Where a, b, c and d where not all zero.

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

7 Upper Triangular Matrix

A square matrix in which all the entries below the principal diagonal elements are zero is known as Upper Triangular matrix.

$$D=\begin{pmatrix} \mathbf{4} & 2 & 1 & 0 \\ 0 & \mathbf{3} & 5 & 10 \\ 0 & 0 & \mathbf{0} & 20 \\ 0 & 0 & 0 & \mathbf{2} \end{pmatrix}$$

$$H=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 20 & 5 & 0 & 0 \\ 10 & -36 & 4 & 0 \\ 9 & 7 & 6 & 14 \end{bmatrix}$$

OR

Let A= [a_{ij}]$_{n\times n}$ where i , j = 1 , 2, 3, ... n.

A is said to be **Upper Triangular Matrix if a_{ij}= 0 ∀ $i > j$**

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} & a_{1n} \\ & a_{22} & \cdots & a_{2(n-1)} & a_{2n} \\ & & \cdots & a_{3(n-1)} & a_{3n} \\ & & . & . & . \\ & & & . & . \\ & & & & . \\ & & & & a_{(m-1)n} \\ & & & & a_{nn} \end{bmatrix}$$

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Lower Triangular Matrix

A square matrix in which all the entries above the principal diagonal elements are zero is known as Lower Triangular matrix.

$$L=\begin{pmatrix} 4 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 9 & 10 & 0 & 0 \\ 30 & 0 & -15 & 2 \end{pmatrix}$$

$$X=\begin{bmatrix} 1 & 2 & 3 & -10 \\ 0 & 5 & 25 & 0 \\ 0 & 0 & 4 & 12 \\ 0 & 0 & 0 & 14 \end{bmatrix}$$

OR

Let A= [a_{ij}]$_{m\times m}$ where i , j = 1 , 2, 3, … m.

A is said to be **Lower Triangular Matrix if $\boldsymbol{a_{ij}}$= 0 ∀ $\boldsymbol{i < j}$**

$$A=\begin{bmatrix} a_{11} & & & & \\ a_{21} & a_{22} & & & \\ a_{31} & a_{32} & \cdots & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ a_{(m-1)1} & a_{(m-1)2} & \cdots & & \\ a_{m1} & a_{m2} & \cdots & a_{m(n-1)} & a_{mm} \end{bmatrix}$$

9 Scalar Matrix

A diagonal matrix in which all diagonal entries are identical is called as Scalar matrix.

S= $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$, T= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ A= $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

Let A= [a_{ij}]$_{n\times n}$ where i = 1 , 2, 3, ... m and j= 1, 2, 3, ...n

$$A=\begin{bmatrix} a_{11} & 0 & \dots & 0 & 0 \\ 0 & a_{22} & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & \dots & a_{(n-1)(n-1)} & 0 \\ 0 & 0 & \dots & 0 & a_{nn} \end{bmatrix}$$

Note: scalar matrices are

i. Square matrix

ii. Diagonal matrix (i.e. all entries except principal diagonal elements are zero)

iii. An identity matrix is scalar matrix.

(But All scalar matrices are not an identity matrix)

Where $a_{11} = a_{22} = \dots = a_{(n-1)(n-1)} = a_{nn}$

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Singular matrix

A square matrix having determinant value as Zero is known as Singular Matrix.

OR

Let A= [a_{ij}]$_{m\times m}$ where i , j = 1 , 2, 3, ... m.

if det (A) = 0 then A is said to be Singular matrix.

D= $\begin{pmatrix} \mathbf{6} & \mathbf{30} \\ \mathbf{4} & \mathbf{20} \end{pmatrix}$

Det(D)= 6 * 20 – 30*4

= 120 – 120

= 0

11 Non-Singular matrix

A square matrix having determinant value as Non-Zero number is known as Non-Singular Matrix.

OR

Let A= [a_{ij}]$_{m \times m}$ where i , j = 1 , 2, 3, ... m.

<u>if det (A) $\neq$ 0</u> then A is said to be Non-Singular matrix.

$$X = \begin{bmatrix} -2 & 3 \\ 8 & 0 \end{bmatrix}$$

$|X|$ = (-2) * 0 – 3 * 8

= 0 – 24

= -24

12 **Transpose of a matrix**

A transpose of a matrix is obtained by interchanging row into column (or column into row) of a matrix.

It is denoted by A' or A^T or A^t

$$A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \\ -5 & 0 \end{pmatrix}_{3\times 2}$$

Transpose is obtained

After interchanging row into column

$$A' = \begin{pmatrix} 2 & 1 & -5 \\ 7 & 3 & 0 \end{pmatrix}_{2\times 3}$$

$$X = \begin{bmatrix} -2 & 3 \\ 8 & 0 \end{bmatrix}_{2\times 2}$$

Transpose is obtained

After interchanging row into column

$$X' = \begin{bmatrix} -2 & 8 \\ 3 & 0 \end{bmatrix}_{2\times 2}$$

13 Symmetric Matrix

Let A= [a_{ij}]$_{m\times n}$ where i = 1 , 2, 3, ... m and j= 1, 2, 3, ...n

A is said to be Symmetric matrix if a_{ij} = a_{ji} ∀ i, j

i. e. if A= A' the A is symmetric matrix

A= $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ then A' = $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

X= $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 7 & -5 \end{bmatrix}$ then X' = $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 7 & -5 \end{bmatrix}$

A= A' , hence A is symmetric matrix.

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Skew- Symmetric Matrix

Let A= [a_{ij}]$_{m\times n}$ where i = 1 , 2, 3, ... m and j= 1, 2, 3, ...n

A is said to be Skew-Symmetric matrix if a_{ij} = - a_{ji} ∀ i, j

i. e. if A= $-$ A$'$ the A is Skew-symmetric matrix

P = $\begin{bmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{-2} & \mathbf{0} \end{bmatrix}$ then P$'$ = $\begin{bmatrix} \mathbf{0} & \mathbf{-2} \\ \mathbf{2} & \mathbf{0} \end{bmatrix}$ $\Rightarrow$ - P$'$ = $\begin{bmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{-2} & \mathbf{0} \end{bmatrix}$

Here **P** = $-$P$'$

Therefore P is skew-symmetric matrix.

15 **Orthogonal Matrix**

A matrix A is said to be orthogonal matrix if product of the matrix and it's transpose is Identity Matrix

i.e. if $A * A' = I$ then A is Orthogonal matrix

If $P = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$ then $P' = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$$P * P' = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} cos^2(\theta) + sin^2(\theta) & -\cos(\theta)\sin(\theta) + \sin(\theta)\cos(\theta) \\ -\sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta) & sin^2(\theta) + cos^2(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Hence P is orthogonal matrix.

Other example $\left(\frac{1}{9}\right)\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$

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Hermitian Matrix

A matrix A is said to be Hermitian matrix if $A = A^{\theta}$ where

A^{θ} is transpose of conjugate of matrix A.

Let A= $\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

Conjugate of A = $\bar{A}$ = $\begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix}$

Transpose of conjugate of A = $\bar{A}'$ = $\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

A^{θ} = $\bar{A}'$ = $\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

Here A= A^{θ}

Therefore A is Hermitian matrix.

17 Skew- Sermitian Matrix

A matrix A is said to be Skew-Hermitian matrix if $A = -A^{\theta}$ where A^{θ} is transpose of conjugate of matrix A.

Let A= $\begin{bmatrix} i & -2 \\ 2 & 0 \end{bmatrix}$

Conjugate of A = $\bar{A}$ = $\begin{bmatrix} -i & -2 \\ 2 & 0 \end{bmatrix}$

Transpose of conjugate of A = $\bar{A}'$ = $\begin{bmatrix} -i & 2 \\ -2 & 0 \end{bmatrix}$

$$A^{\theta} = \bar{A}' = \begin{bmatrix} -i & 2 \\ -2 & 0 \end{bmatrix}$$

$$A^{\theta} = - \begin{bmatrix} i & -2 \\ 2 & 0 \end{bmatrix}$$

Here A = - A^{θ}

Therefore A is Skew- Hermitian matrix.

18 Unitary matrix

matrix A is said to be Unitary Matrix if $A * A^{\theta} = I$

Let $X = \left(\frac{1}{2}\right)\begin{bmatrix}1+i & 1-i\\ 1-i & 1+i\end{bmatrix}$

Then $X^{\theta} = \left(\frac{1}{2}\right)\begin{bmatrix}1-i & 1+i\\ 1+i & 1-i\end{bmatrix}$

$$X * X^{\theta} = \left(\frac{1}{2}\right)\begin{bmatrix}1+i & 1-i\\ 1-i & 1+i\end{bmatrix} * \left(\frac{1}{2}\right)\begin{bmatrix}1-i & 1+i\\ 1+i & 1-i\end{bmatrix}$$

$$= (1/4)\begin{bmatrix}(1+i)*(1-i)+(1-i)*(1+i) & (1+i)^2-(1-i)^2\\ (1-i)^2-(1+i)^2 & (1-i)*(1+i)+(1+i)*(1-i)\end{bmatrix}$$

$$= (1/4)\begin{bmatrix}1^2 - i^2 + 1^2 - i^2 & 1+2i+i^2+1-2i+i^2\\ 1-2i+i^2+1+2i+i^2 & 1^2 - i^2 + 1^2 - i^2\end{bmatrix}$$

$$= (1/4)\begin{bmatrix}1-(-1)+1-(-1) & 2+2i^2\\ 2+2\,i^2 & 1-(-1)+1-(-1)\end{bmatrix}$$

$$= (1/4)\begin{bmatrix}4 & 2-2\\ 2-2 & 4\end{bmatrix}$$

$$= (1/4)\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$