





# Programme: B. Sc. I. T. Course: Applied Mathematics Topic: Matrices

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An arrangements of an element in particular rows and columns is known as matrix

Where elements of matrix are enclosed in parenthesis () or [].

The matrix is denoted by upper case letters.

OR:

An array of dimension  $m \times n$  is known as Matrix where m is number of rows (horizontally lines) and n is number of column (vertical lines)

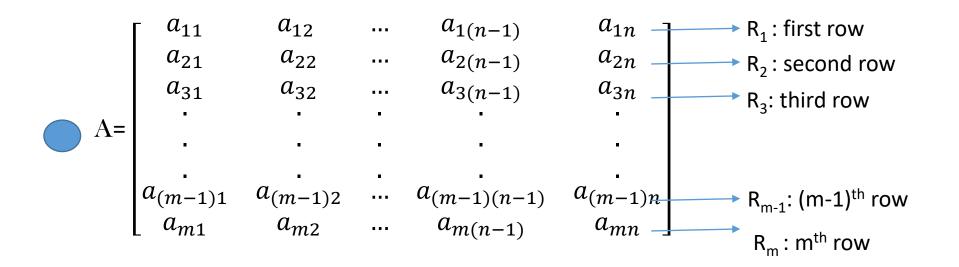
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \\ -5 & 0 \end{pmatrix} \qquad X = \begin{bmatrix} -2 & 3 & 5 \\ 8 & 0 & 4 \\ 12 & 52 & 7 \end{bmatrix}$$

## Let $A^{=}$ [ $a_{ij}$ ]<sub>*m*×*n*</sub> where i = 1, 2, 3, ... m and j= 1, 2, 3, ....n

 $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1(n-1)} & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2(n-1)} & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3(n-1)} & a_{3n} \\ & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ a_{m-1)1} & a_{(m-1)2} & \dots & a_{(m-1)(n-1)} & a_{(m-1)n} \\ a_{m1} & a_{m2} & \dots & a_{m(n-1)} & a_{mn} \end{bmatrix}$ A=

#### Note:

If  $m \neq n$  then matrix A is Rectangular matrix. If m = n then matrix A is Square matrix.



A(i, j): a<sub>ij</sub> : elemment present at ith row and jth column

Order of matrix is  $m \times n$ 

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#### 1 Rectangular Matrix

 $A = \begin{pmatrix} 2 & 7\\ 1 & 3\\ -5 & 0 \end{pmatrix}$ 

Number of rows= 3 Number of column = 2 Order of matrix A is 3 × 2

2 Square Matrix  $X = \begin{bmatrix} -2 & 3 & 5 \\ 8 & 0 & 4 \\ 12 & 52 & 7 \end{bmatrix}$ Number of rows= 3

Number of column = 3 Order of matrix X is  $3 \times 3$  A matrix of order m × n is known as Rectangular matrix.

(Number of rows and column are distinct

i.e. m ≠ *n*.)

 $X = \begin{pmatrix} p & q & r \\ s & t & u \end{pmatrix}$ 

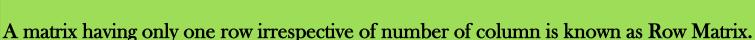
Number of rows= 2 Number of column = 3 Order of matrix X is 2 × 3

A matrix of order  $m \times n$  where m = n is known as Square matrix.

(Number of rows and number of columns are same)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Number of rows= 2 Number of column = 2 Order of matrix A is 2 × 2



 $P = (5 \quad 7)$ Number of rows= 1
Number of column = 2
Order of matrix P is  $1 \times 2$ 

**Column Matrix** 

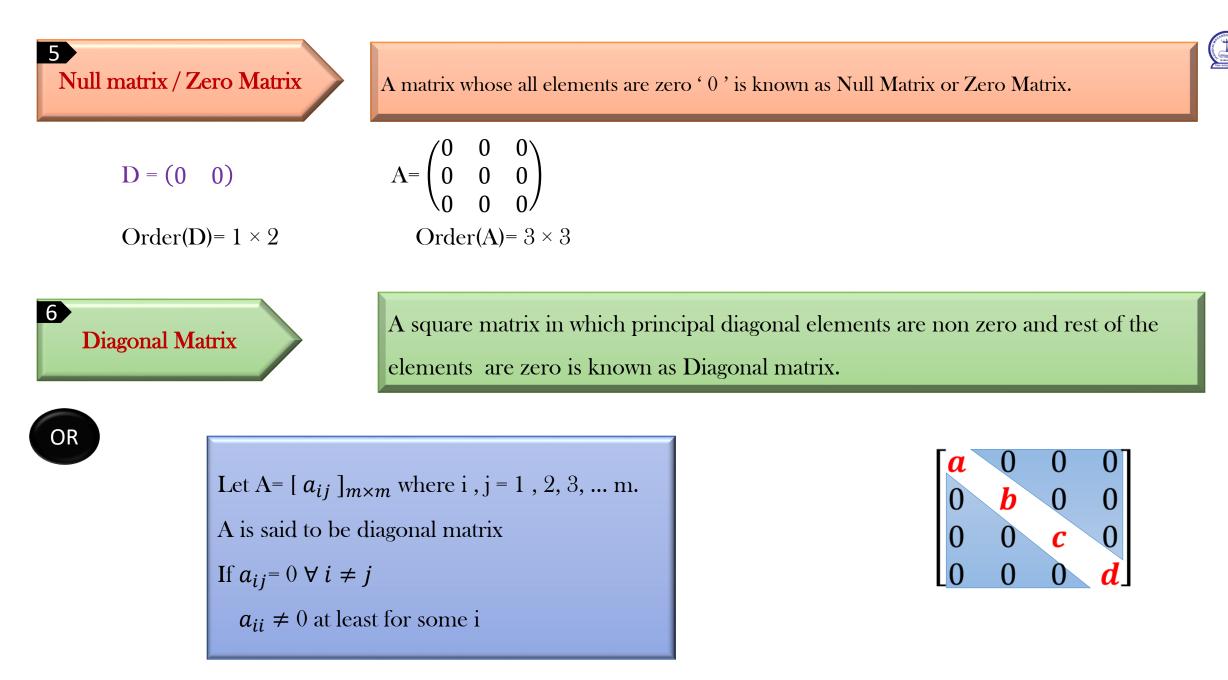
 $Q = [10 \ 25 \ -15 \ 87]$ 

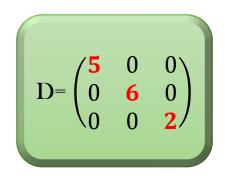
Number of rows= 1 Number of column = 4 Order of matrix Q is 1 × 4

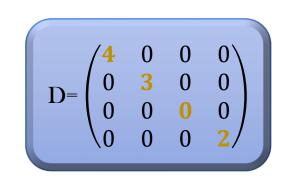
A matrix having only one column irrespective of number of rows is known as Column Matrix.

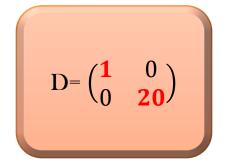
S=

Number of rows= 3 Number of column = 1 Order of matrix S is 3 × 1  $T = \begin{pmatrix} 3 \\ 5 \\ 8 \\ 0 \\ 20 \end{pmatrix}$ Number of rows= 5 Number of column = 1 Order of matrix T is 5 × 1





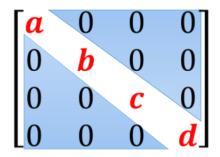




## NOTE:

- i. Matrix is square
- ii. All entries above the diagonal and below the diagonal must be zero '0'.
- iii. Principal diagonal entries are non zero.

Where a, b, c and d where not all zero.





7

A square matrix in which all the entries below the principal diagonal elements are

zero is known as Upper Triangular matrix.

$$B = \begin{pmatrix} 4 & 2 & 1 & 0 \\ 0 & 3 & 5 & 10 \\ 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

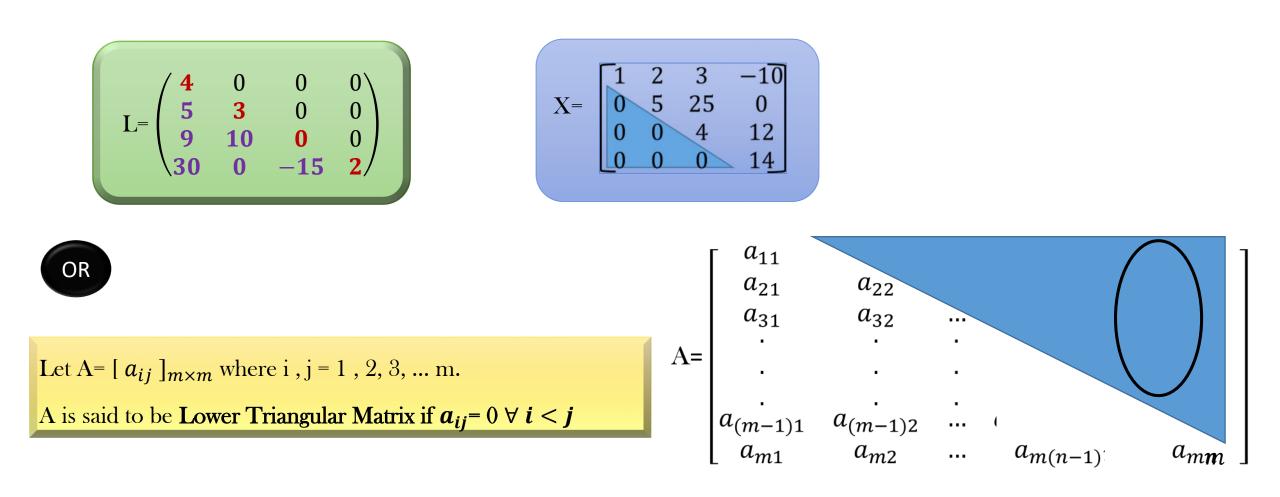
$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 20 & 5 & 0 & 0 \\ 10 & -36 & 4 & 0 \\ 9 & 7 & 6 & 14 \end{pmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} & a_{1n} \\ a_{22} & \cdots & a_{2(n-1)} & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{3(n-1)} & a_{3n} \\ \cdots & \cdots & \cdots \\ a_{(m-1)n} \\ a_{m} \end{bmatrix}$$



A square matrix in which all the entries above the principal diagonal elements are

zero is known as Lower Triangular matrix.



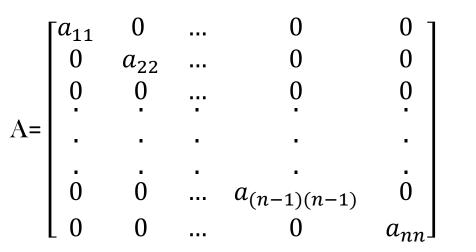


11

#### A diagonal matrix in which all diagonal entries are identical is called as Scalar matrix.

$$S = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} , T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

Let A=  $[a_{ij}]_{n \times n}$  where i = 1, 2, 3, ... m and j= 1, 2, 3, ....n



9

Scalar Matrix

#### Note: scalar matrices are

- i. Square matrix
- ii. Diagonal matrix (i.e. all entries except principal diagonal elements are zero)
- iii. An identity matrix is scalar matrix.
- (But All scalar matrices are not an identity matrix)

Where  $a_{11} = a_{22} = ... = a_{(n-1)(n-1)} = a_{nn}$ 

#### 10 Singular matrix





Let A= 
$$[a_{ij}]_{m \times m}$$
 where i, j = 1, 2, 3, ... m.  
if det (A) = 0 then A is said to be Singular matrix.

$$D = \begin{pmatrix} 6 & 30 \\ 4 & 20 \end{pmatrix}$$
$$Det(D) = 6 * 20 - 30*4$$
$$= 120 - 120$$
$$= 0$$



A square matrix having determinant value as Non-Zero number is known as Non-Singular Matrix.



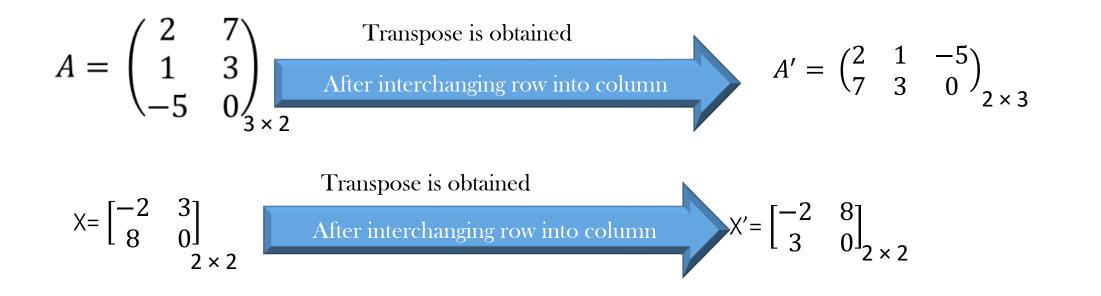
Let A=  $[a_{ij}]_{m \times m}$  where i, j = 1, 2, 3, ... m. <u>if det (A)  $\neq 0$ </u> then A is said to be Non-Singular matrix.

$$X = \begin{bmatrix} -2 & 3 \\ 8 & 0 \end{bmatrix} \qquad |X| = (-2) * 0 - 3 * 8$$
$$= 0 - 24$$
$$= -24$$

#### Transpose of a matrix

12

A transpose of a matrix is obtained by interchanging row into column (or column into row) of a matrix. It is denoted by A' or  $A^T$  or  $A^t$ 







Let A=  $[a_{ij}]_{m \times n}$  where i = 1, 2, 3, ... m and j= 1, 2, 3, ....n A is said to be Symmetric matrix if  $a_{ij} = a_{ji} \forall i, j$ i. e. if A = A' the A is symmetric matrix

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ then } A' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$
$$X = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 7 & -5 \end{bmatrix} \text{ then } X' = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 7 & -5 \end{bmatrix}$$

A=A', hence A is symmetric matrix.





Let  $A = [a_{ij}]_{m \times n}$  where i = 1, 2, 3, ... m and j = 1, 2, 3, ... nA is said to be Skew-Symmetric matrix if  $a_{ij} = -a_{ji} \forall i, j$ i. e. if A = -A' the A is Skew-symmetric matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{0} & \mathbf{2} \\ -\mathbf{2} & \mathbf{0} \end{bmatrix} \text{ then } \mathbf{P}' = \begin{bmatrix} \mathbf{0} & -\mathbf{2} \\ \mathbf{2} & \mathbf{0} \end{bmatrix} \implies -\mathbf{P}' = \begin{bmatrix} \mathbf{0} & \mathbf{2} \\ -\mathbf{2} & \mathbf{0} \end{bmatrix}$$

Here  $\mathbf{P} = -\mathbf{P}'$ 

Therefore P is skew-symmetric matrix.



A matrix A is said to be orthogonal matrix if product of the matrix and it's transpose

is Identity Matrix

i.e. if A \* A' = I then A is Orthogonal matrix

If 
$$P = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$
 then  $P' = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ 

$$P * P' = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos^{2}(\theta) + \sin^{2}(\theta) & -\cos(\theta)\sin(\theta) + \sin(\theta)\cos(\theta) \\ -\sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta) & \sin^{2}(\theta) + \cos^{2}(\theta) \end{bmatrix}$$

 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I

Hence P is orthogonal matrix.

Other example 
$$\left(\frac{1}{9}\right) \begin{pmatrix} 2 & -2 & 1\\ 1 & 2 & 2\\ 2 & 1 & -2 \end{pmatrix}$$



A matrix A is said to be Hermitian matrix if  $A = A^{\theta}$  where

 $A^{\theta}$  is transpose of conjugate of matrix A.

Let  $A = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$ 

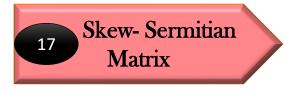
Conjugate of A =  $\overline{A} = \begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix}$ 

Transpose of conjugate of A =  $\overline{A}' = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$ 

$$A^{\theta} = \bar{A}' = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$$

Here  $A = A^{\theta}$ 

Therefore A is Hermitian matrix.



A matrix A is said to be Skew-Hermitian matrix if  $A = -A^{\theta}$  where

 $A^{\theta}$  is transpose of conjugate of matrix A.

Let  $A = \begin{bmatrix} i & -2 \\ 2 & 0 \end{bmatrix}$ 

Conjugate of A = 
$$\overline{A} = \begin{bmatrix} -i & -2 \\ 2 & 0 \end{bmatrix}$$

Transpose of conjugate of A = 
$$\overline{A}' = \begin{bmatrix} -i & 2 \\ -2 & 0 \end{bmatrix}$$

 $A^{\theta} = \bar{A}' = \begin{bmatrix} -i & 2\\ -2 & 0 \end{bmatrix}$  $A^{\theta} = -\begin{bmatrix} i & -2\\ 2 & 0 \end{bmatrix}$ 

Here  $A = -A^{\theta}$ 

Therefore A is Skew- Hermitian matrix.



Let  $X = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$ 

18

Then  $X^{\theta} = \begin{pmatrix} \frac{1}{2} \end{pmatrix} \begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$ 

Unitary matrix

 $X * X^{\theta} = \begin{pmatrix} \frac{1}{2} \end{pmatrix} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix} * \begin{pmatrix} \frac{1}{2} \end{pmatrix} \begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$ 

$$= (1/4) \begin{bmatrix} (1+i) * (1-i) + (1-i) * (1+i) & (1+i)^2 - (1-i)^2 \\ (1-i)^2 - (1+i)^2 & (1-i) * (1+i) + (1+i) * (1-i) \end{bmatrix}$$

$$= (1/4) \begin{bmatrix} 1^2 - i^2 + 1^2 - i^2 & 1 + 2i + i^2 + 1 - 2i + i^2 \\ 1 - 2i + i^2 + 1 + 2i + i^2 & 1^2 - i^2 + 1^2 - i^2 \end{bmatrix}$$

$$= (1/4) \begin{bmatrix} 1 - (-1) + 1 - (-1) & 2 + 2i^2 \\ 2 + 2i^2 & 1 - (-1) + 1 - (-1) \end{bmatrix}$$

$$= (1/4) \begin{bmatrix} 4 & 2 - 2 \\ 2 - 2 & 4 \end{bmatrix}$$

$$= (1/4) \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$